

IES VEGA DE TORANZO

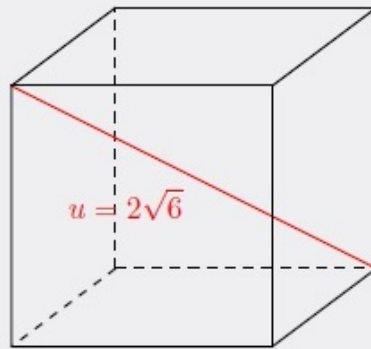
# STEM FOR YOUTH

**EXERCISES OF "GEOMETRY FOR BEGINNERS" AND "PLATONIC SOLIDS"**

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The length of a solid diagonal in a cube is  $u = 2\sqrt{6}$  cm. Find the surface area.



- A  $24\sqrt{2}$  cm<sup>2</sup>
- B  $48$  cm<sup>2</sup>
- C  $24$  cm<sup>2</sup>
- D  $16\sqrt{2}$  cm<sup>2</sup>
- E  $12\sqrt{6}$  cm<sup>2</sup>

$$l^2 + l^2 + l^2 = (2\sqrt{6})^2$$

$$3l^2 = 4 \cdot 6$$

$$l^2 = \frac{24}{3}$$

$$l = \sqrt{8} \text{ cm}$$

The edge of the cube is  $\sqrt{8}$  cm.

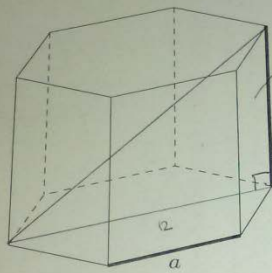
$$S_A = 6 \cdot l^2$$

$$S_A = 6 \cdot 8$$

$$S_A = 48 \text{ cm}^2$$

The surface area is  $48 \text{ cm}^2$ .

The height  $v$  of a regular hexagonal prism is a double of its side  $a$ . The volume of the prism is  $648\sqrt{3} \text{ cm}^3$ . Use this information to find the length of the longest solid diagonal in the prism.



$$c^2 = b^2 + a^2$$

$$a = \sqrt{v^2 + 12^2} = \sqrt{225}$$

$$\sqrt{2 \cdot 12^2} = \sqrt{2 \cdot 144} = 12\sqrt{2}$$

- A  $\sqrt{432} \text{ cm}$
- B  $6\sqrt{10} \text{ cm}$
- C  $10\sqrt{6} \text{ cm}$
- D  $12\sqrt{6} \text{ cm}$
- E  $12\sqrt{2} \text{ cm}$

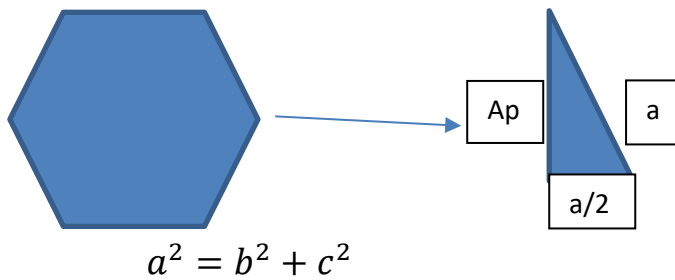
$V = 648\sqrt{3}$   
 $V = A_B \cdot h$   
 $A = \frac{P \cdot a_n}{2}$   
 $P_n \sqrt{3} = \frac{A_B \cdot a}{\sqrt{3}} = \frac{\sqrt{3} \cdot 108^2}{2} = \frac{94\sqrt{3}}{2}$

$(2) AB = \frac{6 \cdot a^2 \sqrt{3}}{2}$   
 $AB = \frac{3a^2 \sqrt{3}}{2}$   
 $648\sqrt{3} = \frac{3a^2 \sqrt{3}}{2} \cdot 2a$   
 $648\sqrt{3} = 3a^3 \sqrt{3}$   
 $\frac{648\sqrt{3}}{3\sqrt{3}} = a^3$   
 $216 = a^3$   
 $a = \sqrt[3]{216} = 6$

Paciosos

Está

Firstly, we calculate the apothem using the Pitagoras' theorem:



$$a^2 = \left(\frac{a}{2}\right)^2 + ap^2$$

$$ap^2 = a^2 - \frac{a^2}{4}$$

$$ap^2 = \frac{3a^2}{4}$$

$$ap = \sqrt{3} \frac{a}{2} \text{ cm}$$

Secondly, we calculate the base area:

$$A_B = \frac{P \cdot ap}{2}$$

$$A_B = \frac{6a \cdot \sqrt{3} \frac{a}{2}}{2}$$

$$A_B = \frac{3\sqrt{3}a^2}{2} \text{ cm}^2$$

Using the expression to calculate the volumen, we obtain the length of the edge of the base:

$$V = A_B \cdot h$$

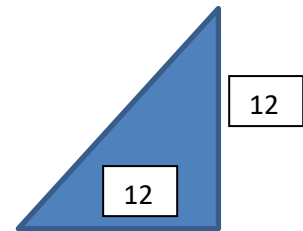
$$648\sqrt{3} = \frac{3\sqrt{3}a^2}{2} 2a$$

$$648\sqrt{3} = 3\sqrt{3}a^3$$

$$216 = a^3$$

$$a = 6 \text{ cm}$$

Finally, the length of the longest solid diagonal is obtained using the Pitagoras' theorem again:



$$a^2 = b^2 + c^2$$

$$d^2 = (2a)^2 + (2a)^2$$

$$d^2 = 12^2 + 12^2$$

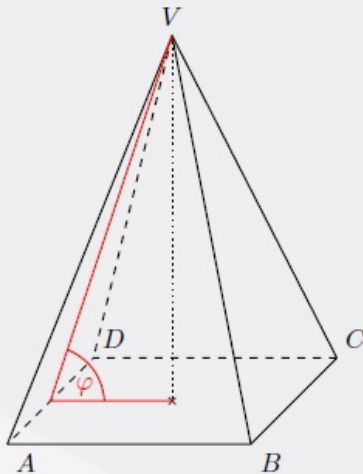
$$d^2 = 144 + 144$$

$$d = \sqrt{288}$$

$$d = 12\sqrt{2} \text{ cm}$$

The length of the longest solid diagonal is  $12\sqrt{2} \text{ cm}$ .

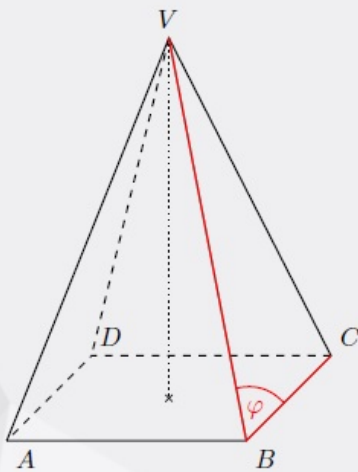
Give a verbal description to the angle shown in the picture.



- A The angle between the edge and the base
- B The angle between the edge of a triangular face and the base edge
- C The angle between the triangular face and the base
- D The angle between two triangular faces

The answer is “the angle between the triangular face and the base”.

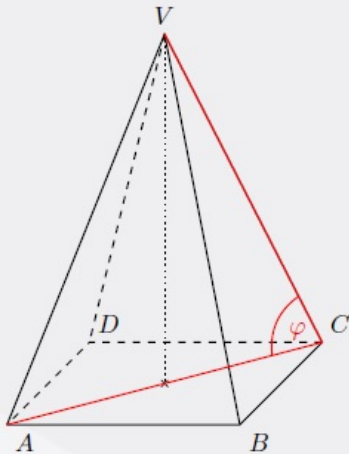
Give a verbal description to the angle shown in the picture.



- A The angle between two triangular faces having a common edge
- B The angle between the edge on triangular face and the base edge from the same face
- C The angle between a triangular face and square base
- D The angle between the triangular face and a base edge not in this face

The answer is “the angle between the edge on triangular face and the base edge form the same face”.

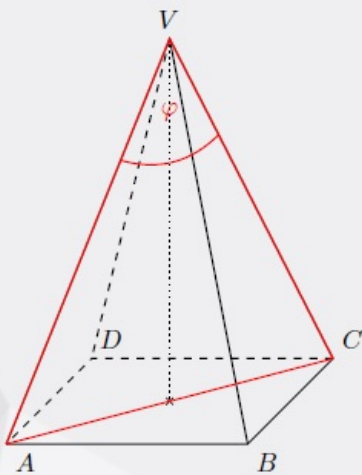
Give a verbal description to the angle shown in the picture.



- A The angle between the edge and the base
- B The angle between two opposite edges
- C The angle between the edge on a face and the base edge
- D The angle between the triangular face and the square base

The answer is “the angle between the edge and the base”.

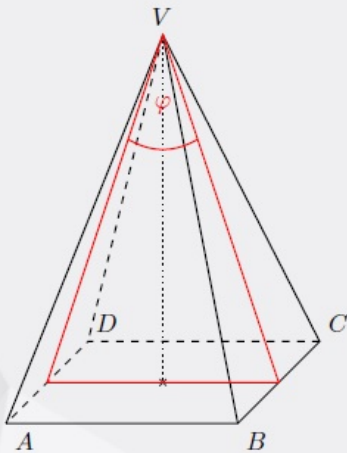
Give a verbal description to the angle shown in the picture.



- A The angle between a triangular face and an edge from the opposite triangular face
- B The angle between two triangular faces having a common edge
- C The angle between opposite edges
- D The angle between two opposite triangular faces

The answer is “the angle between opposite edges”.

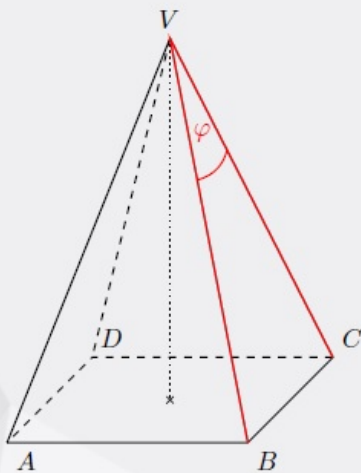
Give a verbal description to the angle shown in the picture.



- A The angle between a triangular face and the base
- B The angle between two edges in the same triangular face
- C The angle between two opposite triangular faces
- D The angle between two triangular faces having a common edge

The answer is “the angle between two opposite triangular faces”.

Give a verbal description to the angle shown in the picture.



- A The angle between two opposite edges
- B The angle between two edges in a common triangular face
- C The angle between two triangular faces having common edge
- D The angle between two opposite triangular faces

The correct answer is “the angle between edges in a common triangular face”.

Given a translation of a plane, find the property of a line obtained by translating a line  $r$ . The line  $r$  is neither parallel nor perpendicular to the translation vector.

- A The resulting line is parallel to the line  $r$
- B The resulting line is perpendicular to the line  $r$
- C The resulting line is the line  $r$ . (The line  $r$  is mapped into itself.)
- D The resulting line is perpendicular to the translation vector.

The answer is “the resulting line is parallel to the line  $r$ ”.

Given a translation  $T$  of a plane, find the lines which are mapped to the same line by  $T$ .

- A All lines perpendicular to the translation vector are mapped into itself.
- B All lines parallel to the translation vector are mapped into itself.
- C Every line is mapped into itself by the translation.
- D There are no lines which are mapped into itself by the translation.

The answer is “all the lines parallel to the translation vector are mapped into itself”.

The dilatation defined by a center and a scale factor  $k = -1$  is equivalent to another geometric mapping. Which one?

- A translation
- B reflection through the line
- C rotation
- D reflection through the point

The answer is “reflection through the point”.



Consider a rotation through an angle either  $\alpha = 180^\circ$  or  $\alpha = 360^\circ$ . How many lines which are mapped into itself exists for this rotation?

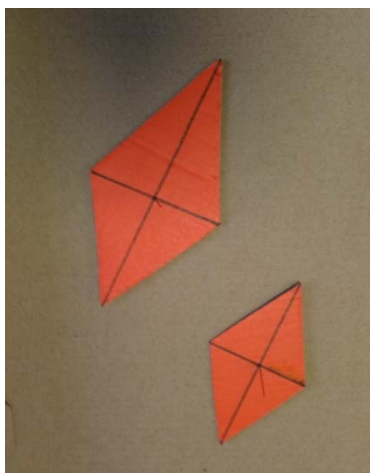
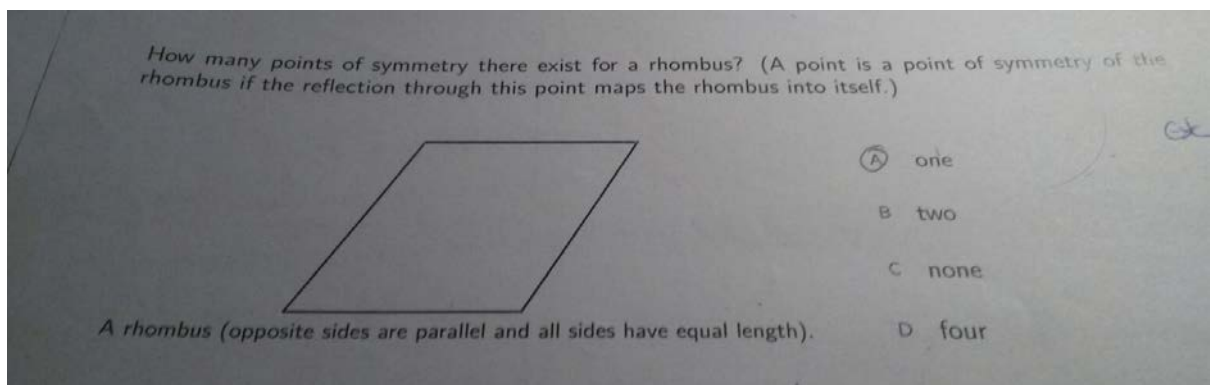
- A infinitely many
- B two
- C none
- D one

The answer is “infinitely many”. In these rotations each line is mapped into itself point by point ( $\alpha = 360^\circ$ ) or semiline by semiline ( $\alpha = 180^\circ$ ).

The rotation through an angle  $\alpha = 180^\circ$  is equivalent to some other geometric mapping. Which one?

- A translation
- B reflection through the line
- C reflection through the point

The answer is “reflection through a point”.

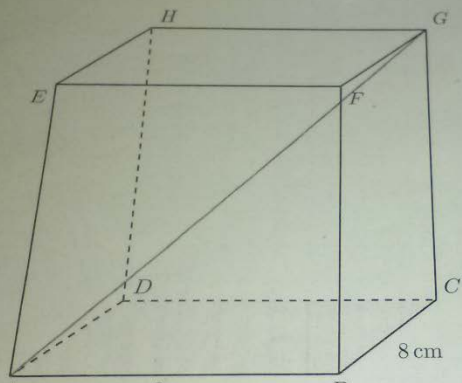


It has just one point of symmetry. The one where the diagonals cut. The reflection through it maps the rhombus into itself.

To see it, we use a pin in the point where the diagonals cut and made the rhombus rotate over it.

The base of a rectangular box  $ABCDEFGH$  has sides  $|AB| = 6\text{ cm}$  and  $|BC| = 8\text{ cm}$ . The angle between the solid diagonal  $AG$  and the base  $ABC$  is  $60^\circ$ . Find the volume of the box.

Give a v

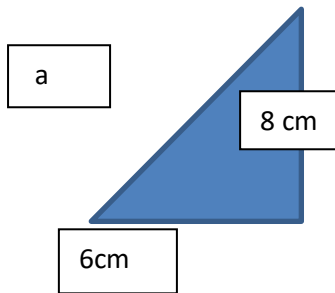


A  $160\sqrt{3}\text{ cm}^3$   $V = A_b \cdot h$   $A_b = 6 \cdot 8 = 48\text{ cm}^2$   
 $V = 48 \cdot h$   
 $V = 48 \cdot 17.32$   
 $V = 831.36\text{ cm}^3$   $\neq$   
 B  $288\sqrt{3}\text{ cm}^3$   
 C  $240\text{ cm}^3$   $\neq$   
 D  $480\sqrt{3}\text{ cm}^3$   
 E  $960\text{ cm}^3$

$a = \sqrt{8^2 + 6^2} = \sqrt{100} = 10\text{ cm}$   
 $\tan 60^\circ = \frac{h}{10}$   
 $10 \tan 60^\circ = h$   
 $h = 17.32$

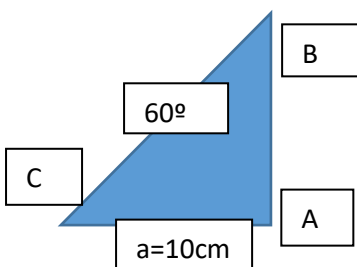
Firstly, we use the Pitagoras' theorem to calculate the length of the base diagonal.

$$a^2 = b^2 + c^2$$



$$a = \sqrt{8^2 + 6^2} = \sqrt{100} = 10\text{ cm}$$

Secondly, we use the definition of tangent to obtain the high of the prism.



$$\tan 60^\circ = \frac{h}{10}$$

$$h = 10 \tan 60^\circ$$

$$h = 10\sqrt{3}\text{ cm}$$

Finally, we calculate the volumen of the prism:

$$V = A_B \cdot h$$
$$V = 6 \cdot 8 \cdot 10\sqrt{3}$$
$$V = 480\sqrt{3}cm^3$$

Consider a dilatation which maps  $A$  onto  $B$ . The center of the dilatation is  $S$ . Find a correct statement.

- A The distance from  $S$  to  $A$  is smaller than the distance from  $A$  to  $B$ .
- B The points  $S$ ,  $A$  and  $B$  form a triangle  $ABS$  with at least two sides of equal length.
- C The point  $S$  is on the line through the points  $A$  and  $B$ .
- D The points  $S$ ,  $A$  and  $B$  form a right triangle  $ABS$ .

The correct answer is “the point  $S$  is on the line through the points  $A$  and  $B$ ”.

Given a line in a plane, find how many points are mapped onto itself with reflection through this line.

- A two
- B infinitely many
- C none
- D one

The correct answer is infinitely many: all the points of the line.



These are others figures that we have used to answer others exercises. We have painted and cut them to build the diferent polyhedra, when we have be able to do that. One example of these exercises is:



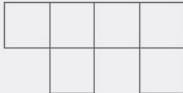
Plato, ancient Greek philosopher, student of Socrates, teacher of Aristotle




For each solid identify a template which can be used to build the paper model of this solid.



**Solids**

1 Pyramid with square base      3 Octahedron (8 faces)      5 Tetrahedron (4 faces)  
 2 Prism with triangle base      4 Cube

**Templates**

a       d       g 

b       e       h 

c       f       i 