Exponential Function

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WHY WE CHOSE EXPONENTIAL FUNCTION?

- On the first look you can see that it combines physics, economy and mathematics, what a strange combination, right?

- On our first meeting we divided tasks by interest:
  - Miranda - economy problem
  - Lourdes - nuclear decay
  - Andjela – capacitor problem, two armies problem and experimental data analyses

❑ Summary

- Mathematics and finances
- ❖ This part: money problem
- Problems in mathematics and banking
- ❖ Banking problems
- Problems in physics and chemistry
- ❖ Conclusions about physics and chemistry problems

- The problems

There were many many difficult problems…

- The hypothesis

We stablished several hypothesis for each problem.

- Testing hypothesis

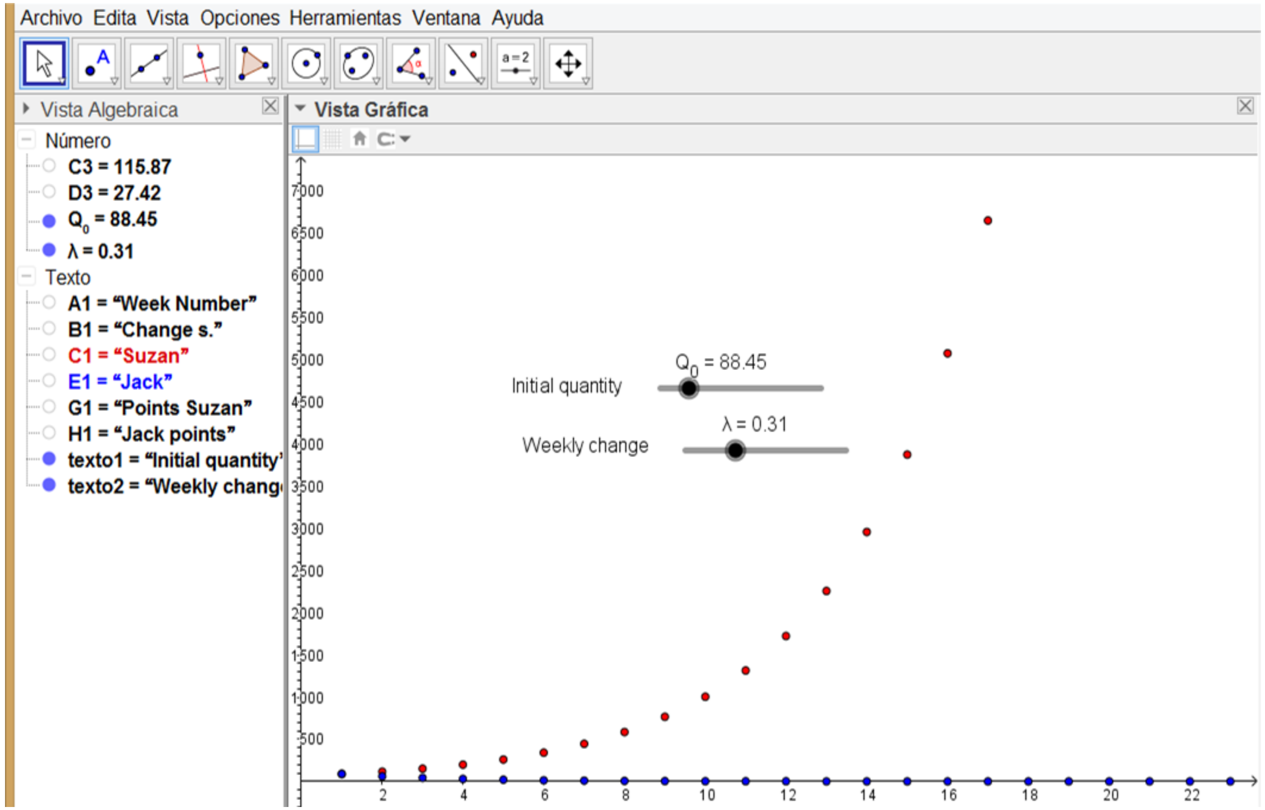
GeoGebra, Open Office… were our **laboratory** to test hypothesis

- Conclusions

We used mathematics to prove solutions and make conclusions

• Mathematics and finances:

This image represents the two graphics of Suzan and Jack, we can see that Suzan's money increases as time goes by, whereas Jack's money decreases.



- Problems in mathematics and banking:

GeoGebra Video ABC

- Problems in mathematics and banking:

GeoGebra Video Jack below100 gcu

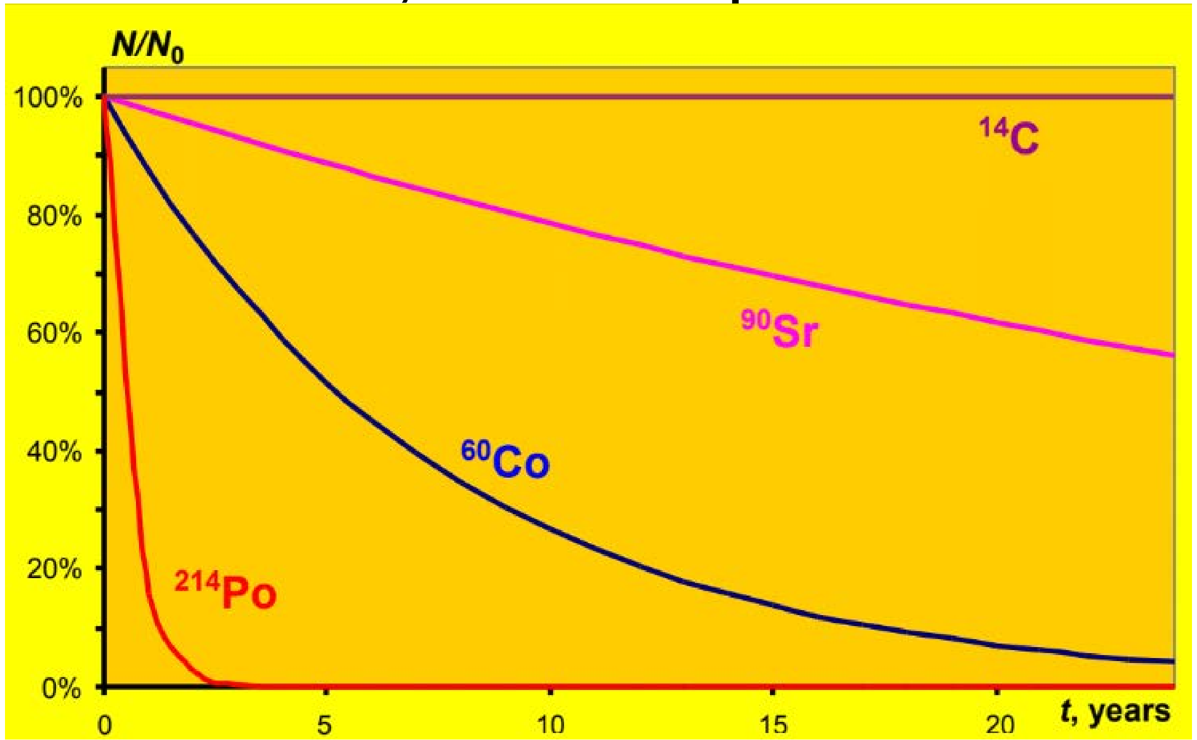
- # Problems in mathematics and banking:

In the question 4, we have reached two final conclusions:

a) Even rounding to 0 decimal places it also gives you the right answer.
b) The decrease is soft and it depends greatly of the interest rate.

- # Problems in physics and chemistry:

This graph shows the decay process of four radioactive isotopes. Using Geogebra and some spreadsheets, we explain the different shapes of the lines on the graph and verify hypothesis about the decay constant λ and *half life*, for example.

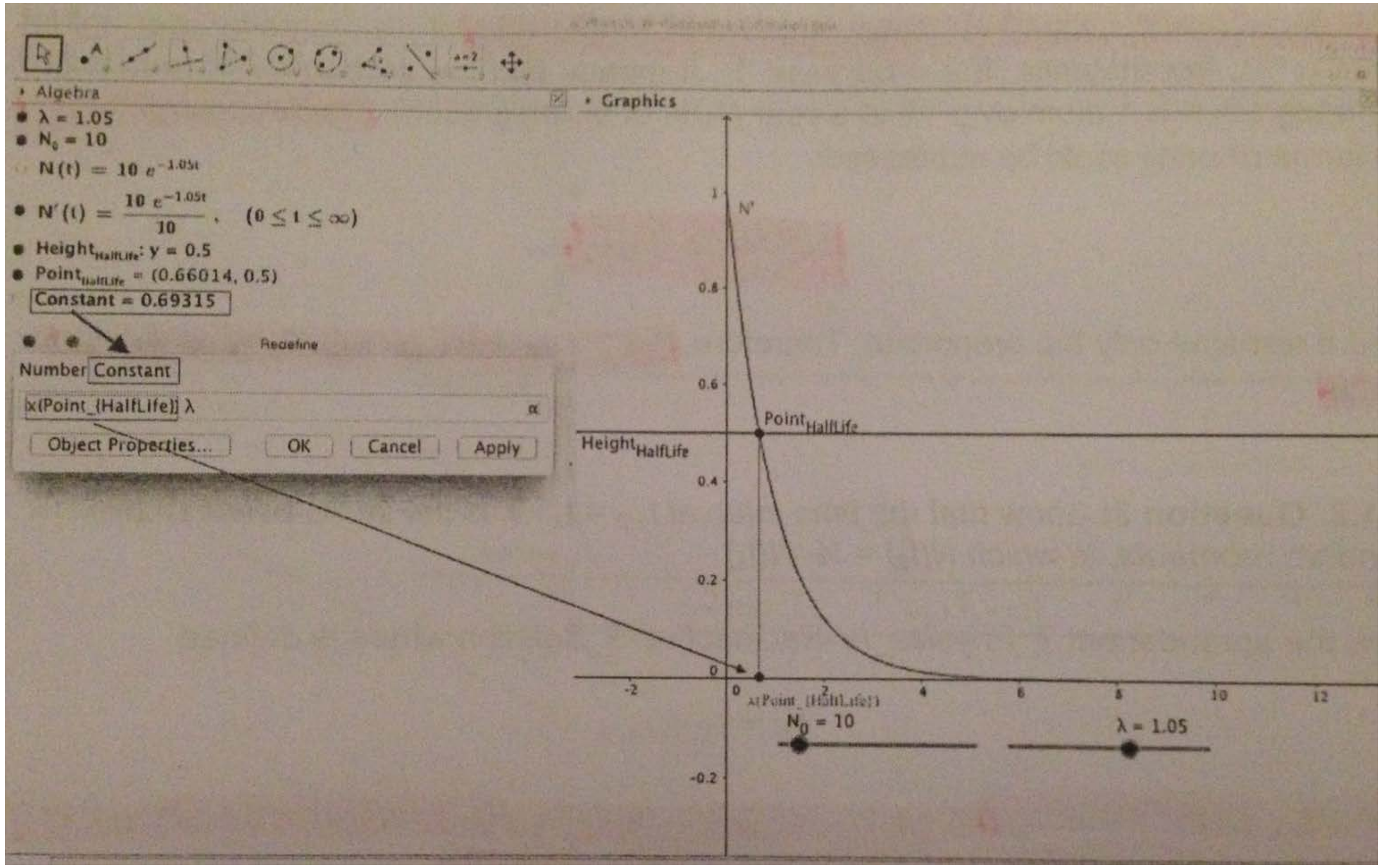


- Problems in physics and chemistry:

GeoGebra Video Half-life

❖ Conclusions about physics and chemistry problems

- ➢ Why constant of *half life* · λ is correct only for $\lambda > 0$ values.
- ➢ Get the 100% efficiency of each isotope using Geogebra sliders.



4.2. CHARGING CAPACITOR PROBLEM

4.2.1.

Ohm's law: $$I = \frac{U - U_c}{R}$$

$$I = \frac{\Delta Q}{\Delta t}$$

$$U_c = \frac{Q}{C}$$

U_c - Voltage at some chosen time t
Q – charge of the capacitor in that moment
C – constant capacitance
U – constant voltage of a power source
Qm – maximum charge

Final equation: $$\Delta Q = \frac{U}{R} \cdot \left(1 - \frac{Q}{UC}\right) \Delta t \longrightarrow Q(t) = UC \cdot \left(1 - e^{-\frac{t}{RC}}\right)$$

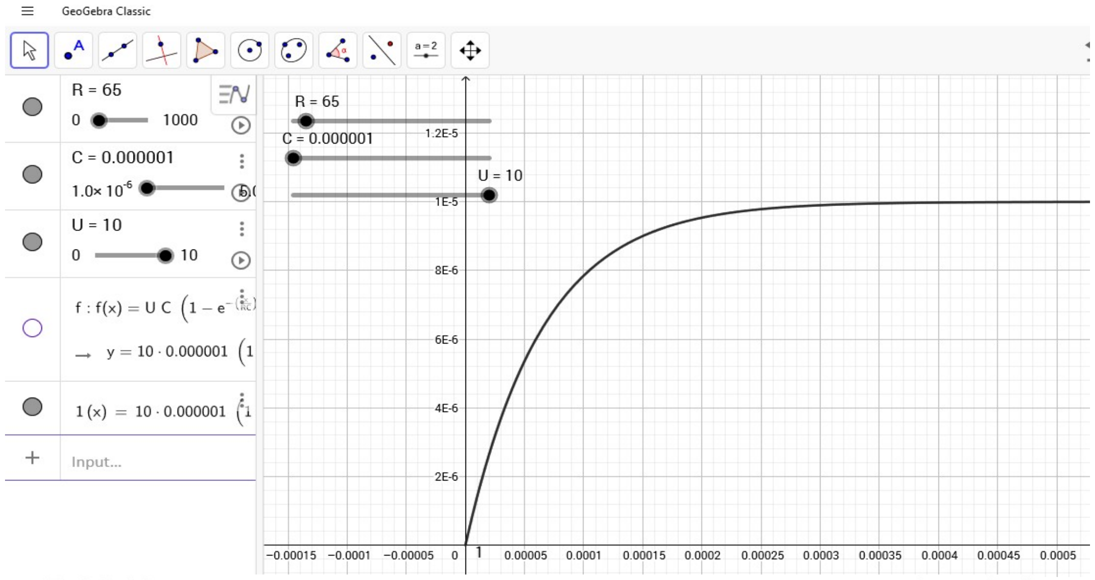
The equation we were looking for: $$\longrightarrow Q(t) = Q_m \cdot \left(1 - e^{-\lambda \cdot t}\right)$$

$Q_m = UC$ and $\lambda = \frac{1}{RC}$

4.2.2. Charging half-time

$$Q_f = Q_m - Q(t)$$

Q_f - missing charge $\qquad Q_f = UC \cdot e^{\frac{-t}{RC}}$



We can define charging half-time $t_{1/2}$ as **time that it will take the capacitor to half the missing charge Q_f.**

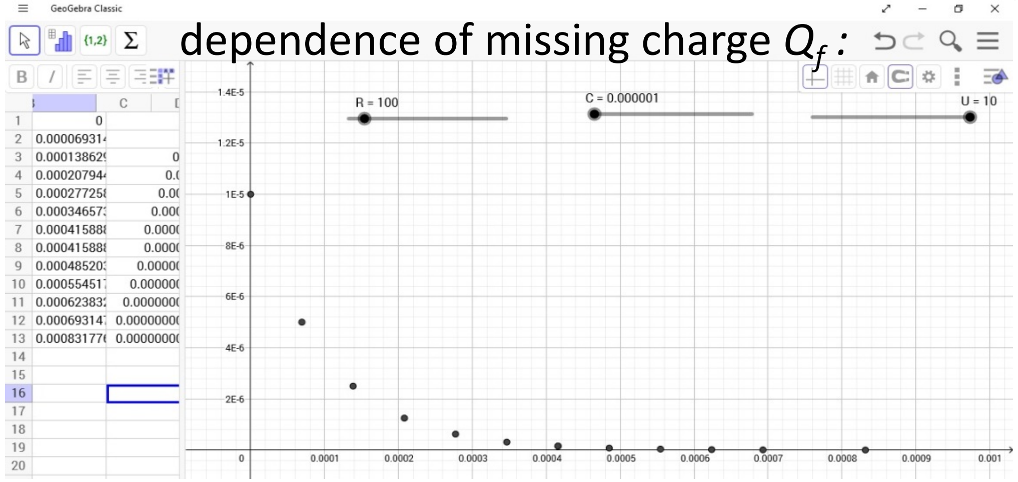
We will now observe Q_{f1} and Q_{f2}, missing charges after periods t_1 and t_2.

$$t_{1/2} = t_2 - t_1 \longrightarrow Q_{f2} = \frac{Q_{f1}}{2}$$

$$UC \cdot e^{\frac{-t1}{RC}} = 2UC \cdot e^{\frac{-t2}{RC}} \longrightarrow \frac{1}{2} = e^{\frac{-t_{1/2}}{RC}}$$

$$\boxed{t_{1/2} = \ln 2 \cdot RC}$$

dependence of missing charge Q_f :



4.2.3. Finding dependence between $\mathbf{Q(t_1)}$ and $\mathbf{Q(t_2)}$ when $\mathbf{t_1}$ and $\mathbf{t_2}$ satisfy $\mathbf{t_2 - t_1 = t_{1/2}}$.

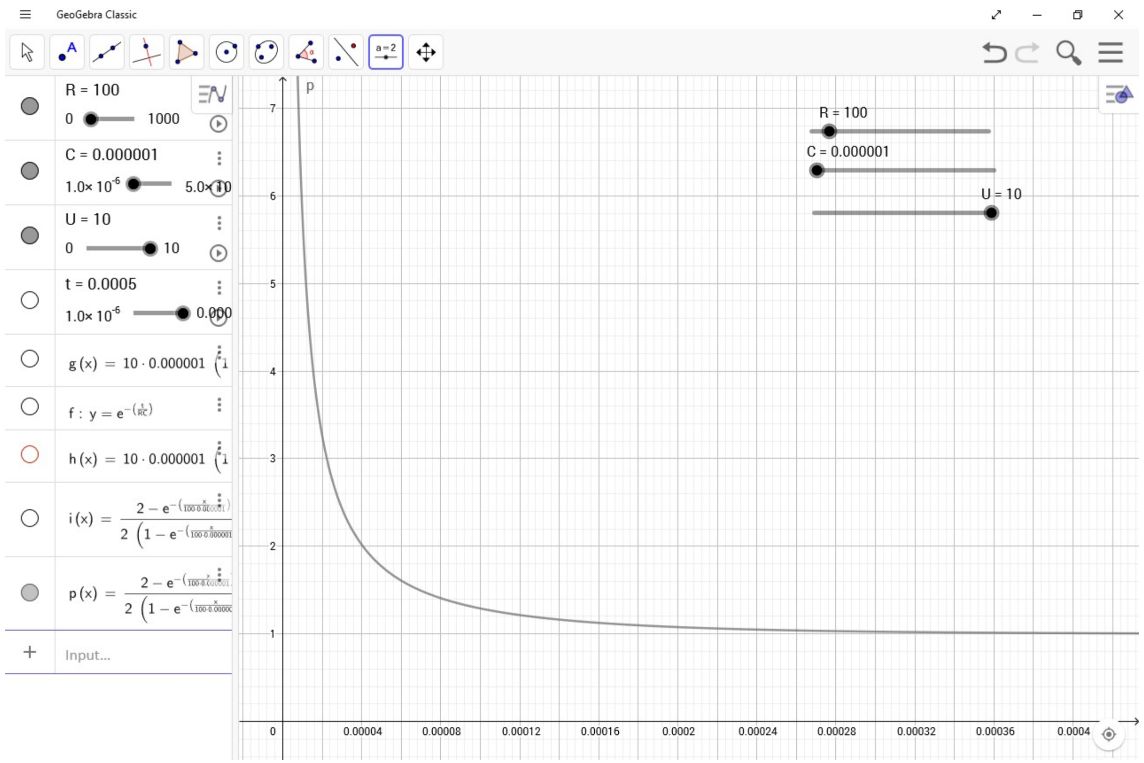
$$K = \frac{Q_2}{Q_1}$$

$$t_2 - t_1 = t_{1/2}, \longrightarrow Q_{f2} = \frac{Q_{f1}}{2}$$

$$UC - Q_1 = 2(UC - Q_2) \longrightarrow 2\frac{Q_2}{Q_1} - 1 = \frac{UC}{Q_1}$$

$$K = \frac{UC}{2Q_1} + \frac{1}{2}$$

$$\boxed{K = \frac{2 - \left(e^{-\frac{t_1}{RC}}\right)}{2\left(1 - e^{-\frac{t_1}{RC}}\right)}}$$



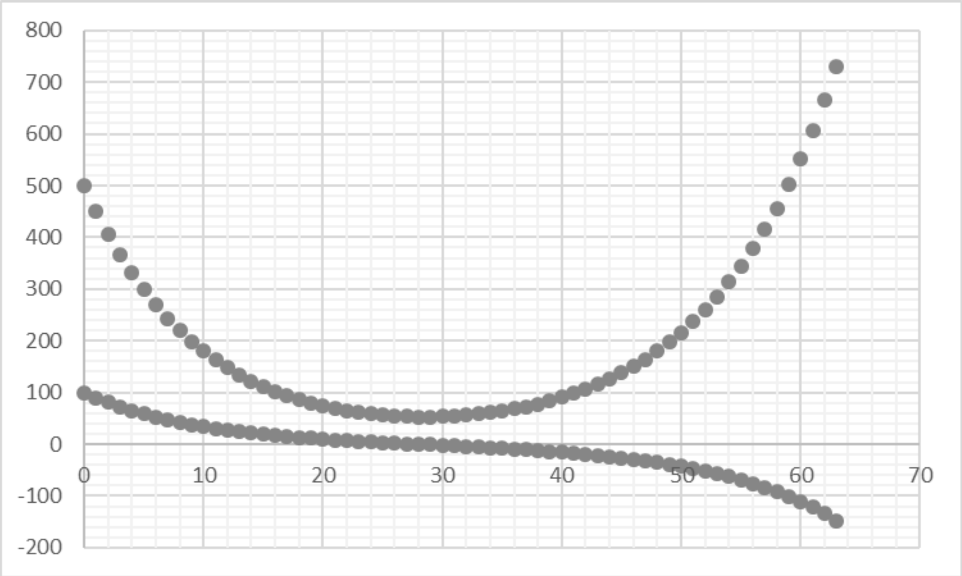
5.1. TWO ARMIES PROBLEM

$\Delta N_1 = -bN_2\Delta t$

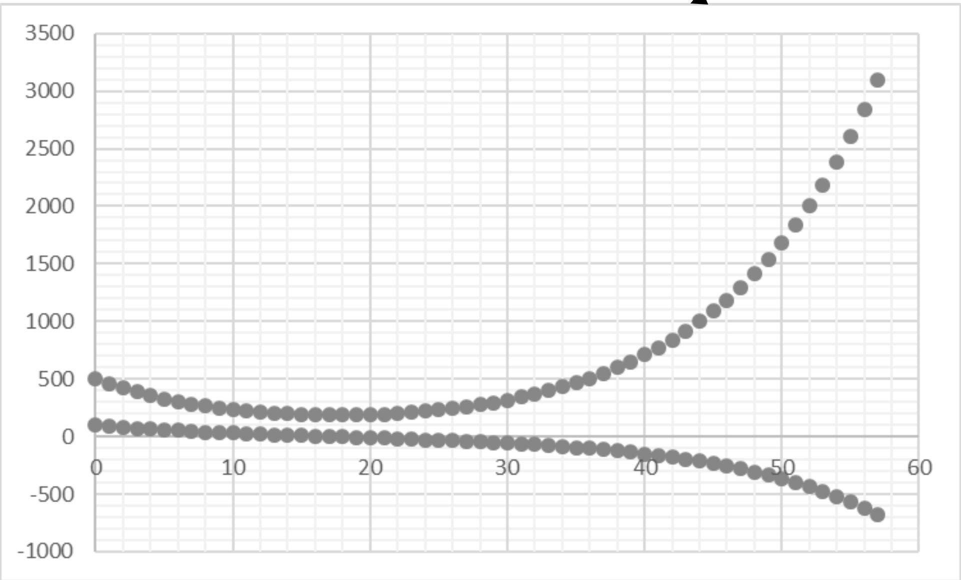
$\Delta N_2 = -aN_1\Delta t$

a - battle ready coefficient for army A
b - battle ready coefficient for army B
Where Δt is time step who is supposed to have small value in order that function can be better simulated.

Can not happen in real life

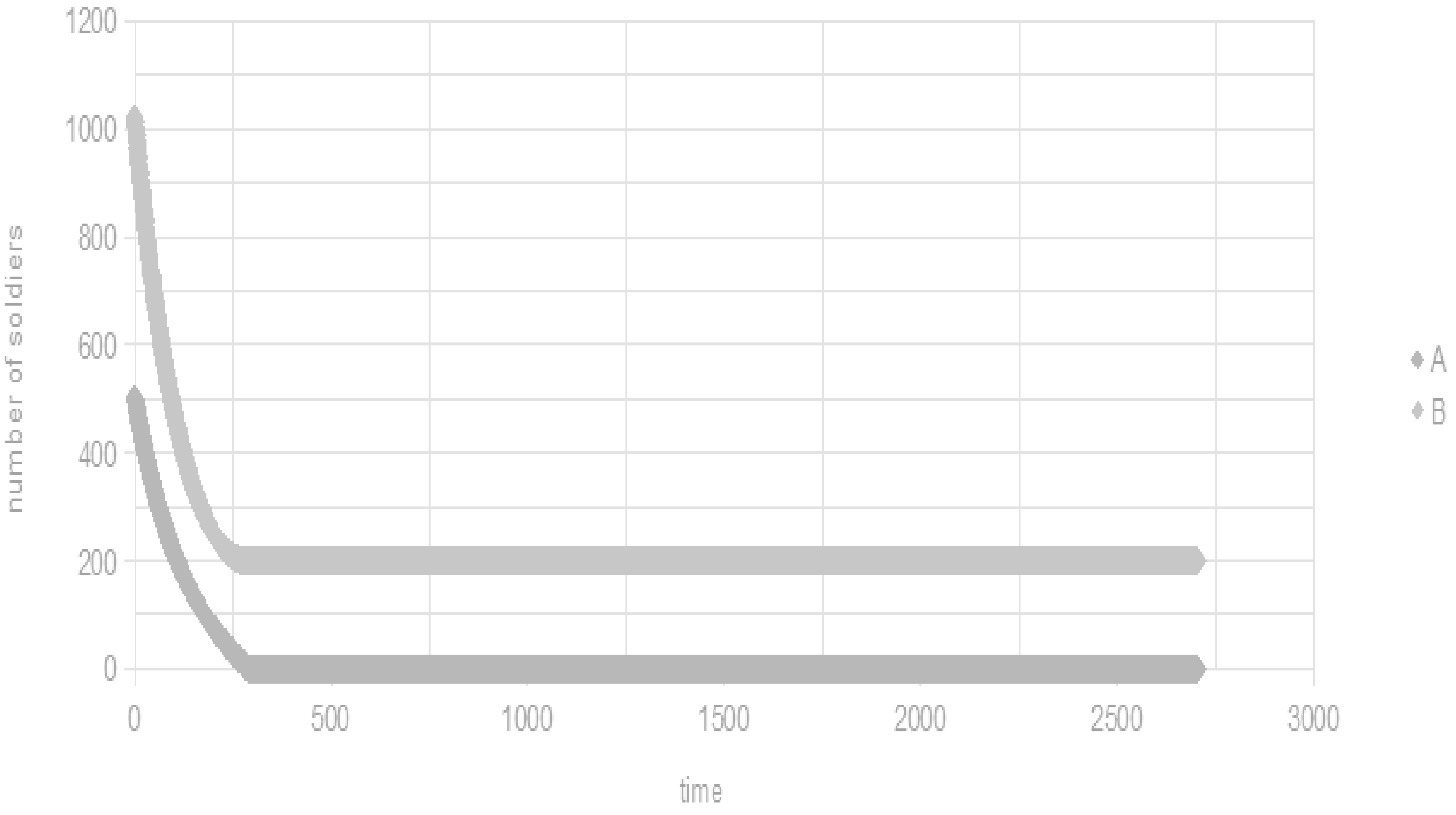


Time evolution of soldiers for N_{0A}=100, N_{0B}=500, *a*=0.4925, b=0.02



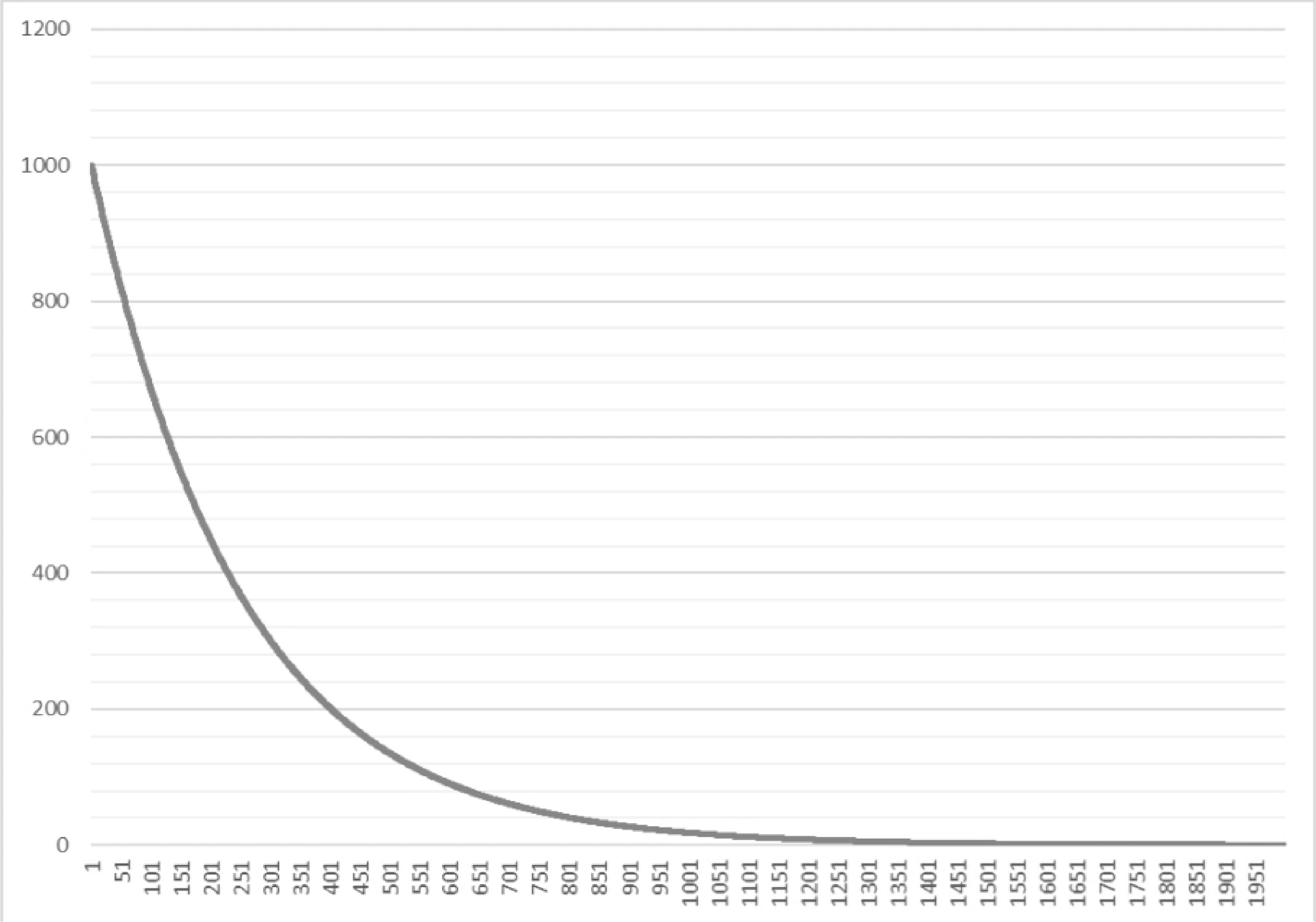
Time evolution of soldiers for N_{0A}=100, N_{0B}=500, a=0.4198, b=0.02.

NOT EXPONENTIAL


number of soldiers
1200
1000
800
600
400
200
0
0
500
1000
1500
2000
2500
3000
time
A
B

5.1.2. When is the function exponential?

Case 1: $N_{0A} = N_{0B}$ & $a = b$

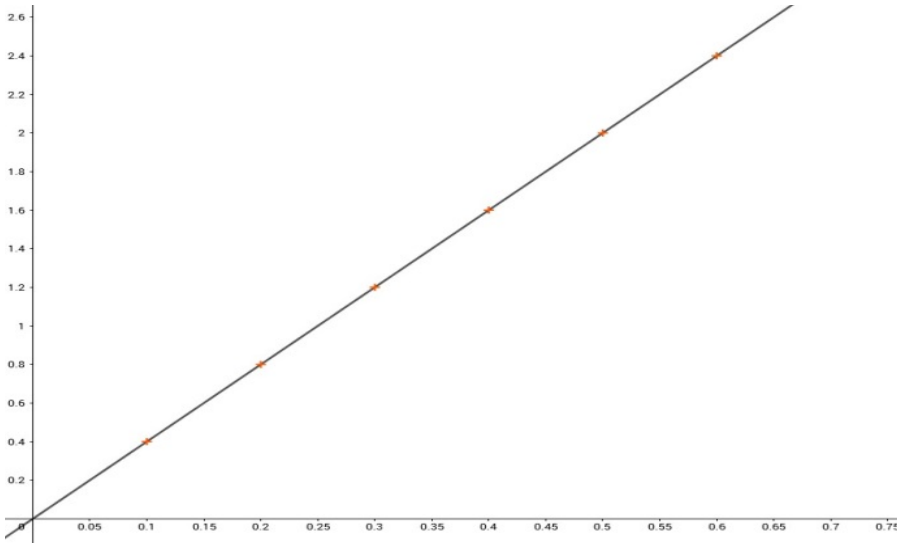


Trivial case where both armies are equal.

Case 2.

N_{0a}/N_{0b}	$a = 0.1$	$a = 0.2$	$a = 0.3$	$a = 0.4$	$a = 0.5$	$a = 0.6$	
2.5	0.625	1.25	1.875	2.5	3.125	3.75	b
2	0.4	0.8	1.2	1.6	2	2.4	
1.5	0.225	0.45	0.675	0.9	1.125	1.35	

Example of the evolution of number of soldiers in both armies, for $N_{0A} = 1000$, N_{0B}=500, $\lambda_a = 0.1$, $\lambda_b = 0.4$



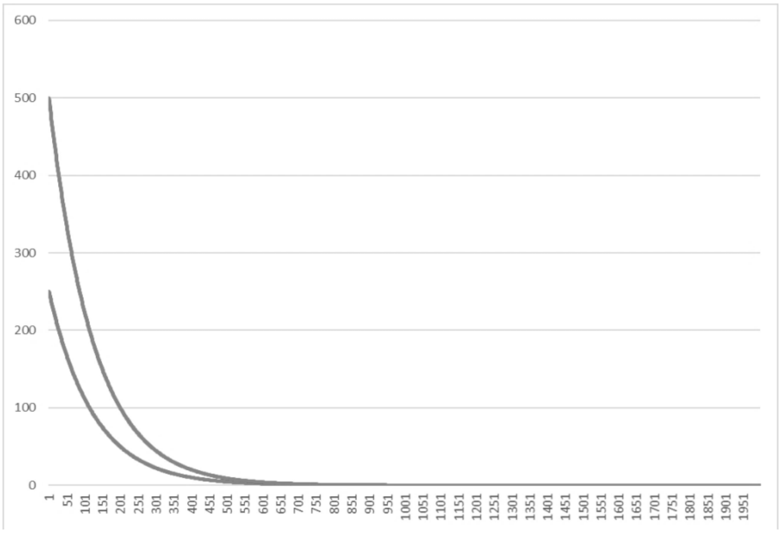
Linear dependence between factors of proportionality

$$\boldsymbol{N_{0A}/N_{0B} = const \rightarrow {}^{a}/_{b} = const}$$

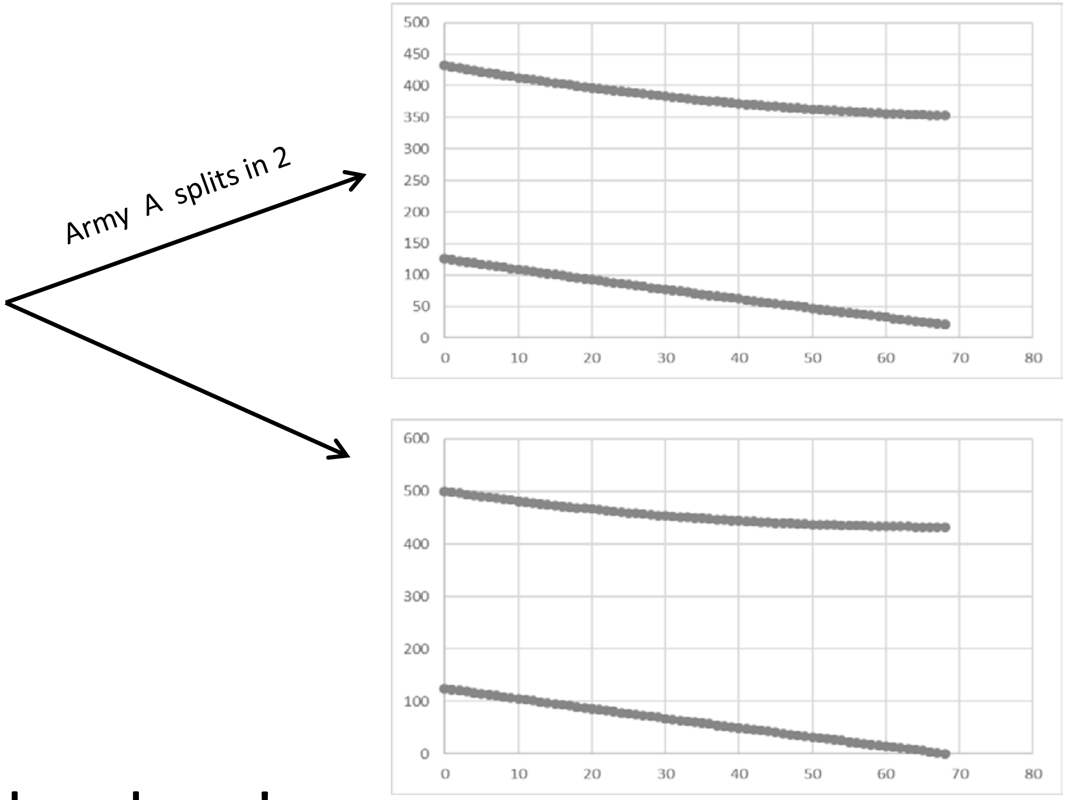
$$\frac{a}{b} = \frac{N_{0B}^2}{N_{0A}^2}$$

5.1.3 What happens when we split one army into two detachments?

- evolution of soldier numbers must be exponential for both armies (showed in the previous)



$N_{0A} = 250, N_{0B} = 500, \lambda_\mathrm{a} = 0.016, \lambda_\mathrm{b} = 0.004$

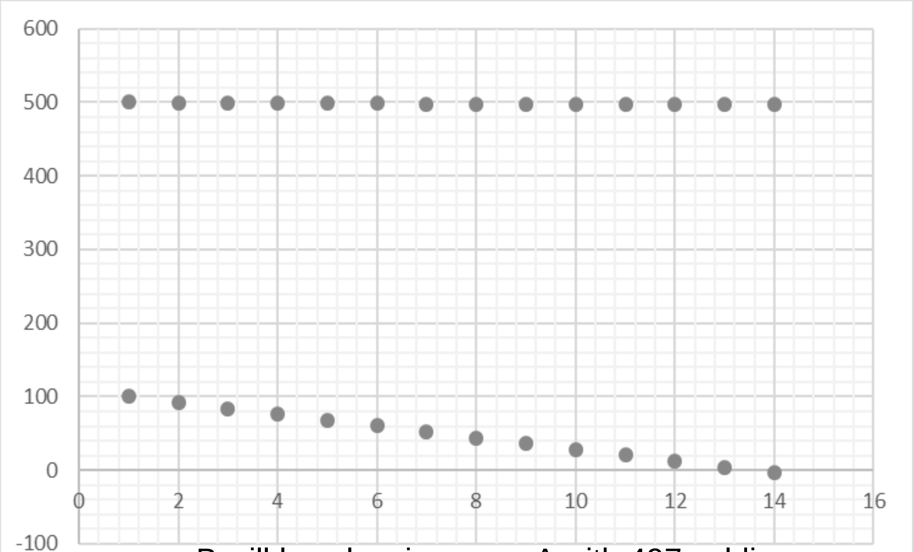


Army who splitted soldiers always looses!

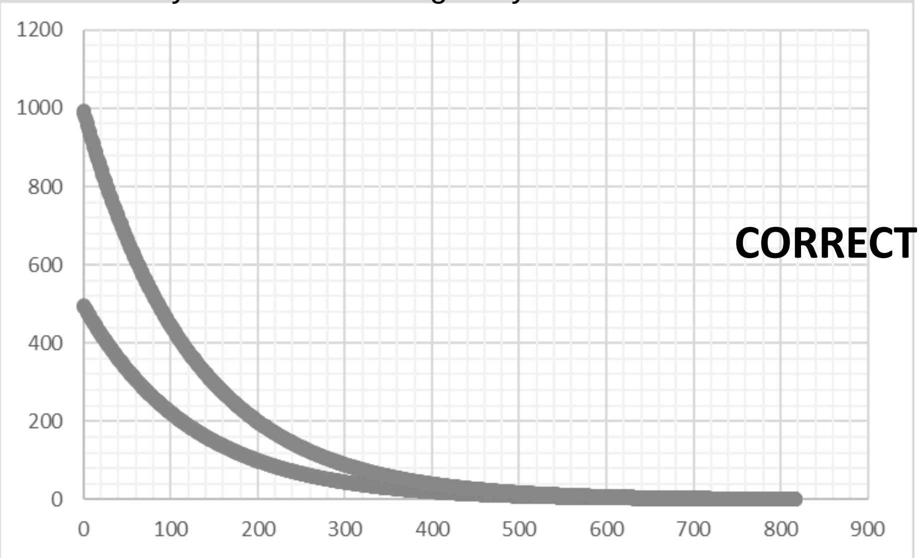
Question if hypothesis $N_{b1}{}^2 + N_{b2}{}^2 = N_{b0}{}^2$ is right.

N_{0A}= 500, N_{0B}= 1000, *a* =0.016, b= 0.004.

1) $N_{b1} = 100$ and $N_{b2} = 994$.



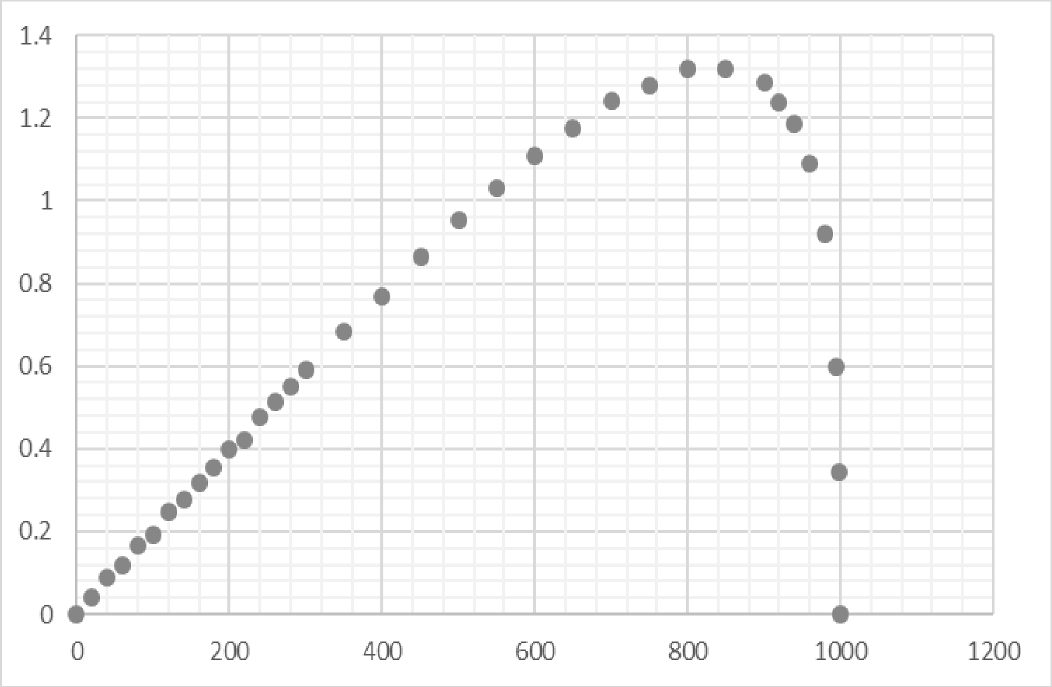
army B will lose leaving army A with 497 soldiers



CORRECT

2) $N_{b1} = 400$ and $N_{b2} \approx 916$

INCORRECT



6. EXPERIMENTAL DATA ANALYSIS

6.1.

*<u>Is the function exponential?</u>

Isaac Newton’s law of Cooling

Hc – heat transfer coefficient (W/K)
T – current temperature (depends on time)
T_r – ambient temperature (constant)

$$\frac{\Delta Q}{\Delta t} = H_c(T - T_r)$$

$$\frac{C \cdot m \cdot \Delta T}{\Delta t} = -H_c(T - T_r)$$

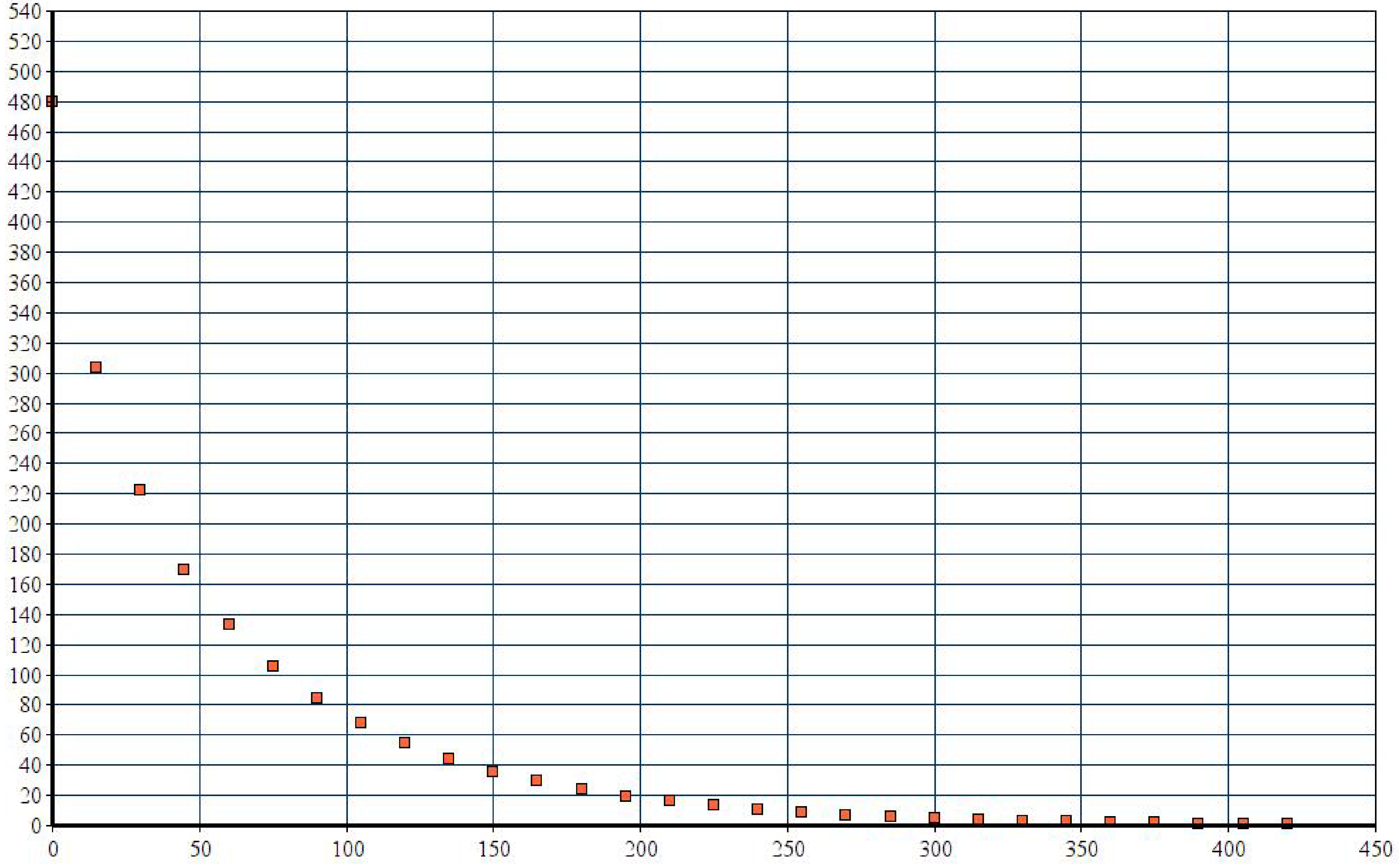
$$\Delta T = -\frac{H_c}{C \cdot m}(T - T_r)\Delta t$$

We can notice that change of $T - T_r$ depends only on the change of *T*, because T_r is constant.

$$\boxed{\Delta(T - T_r) = \Delta T}$$

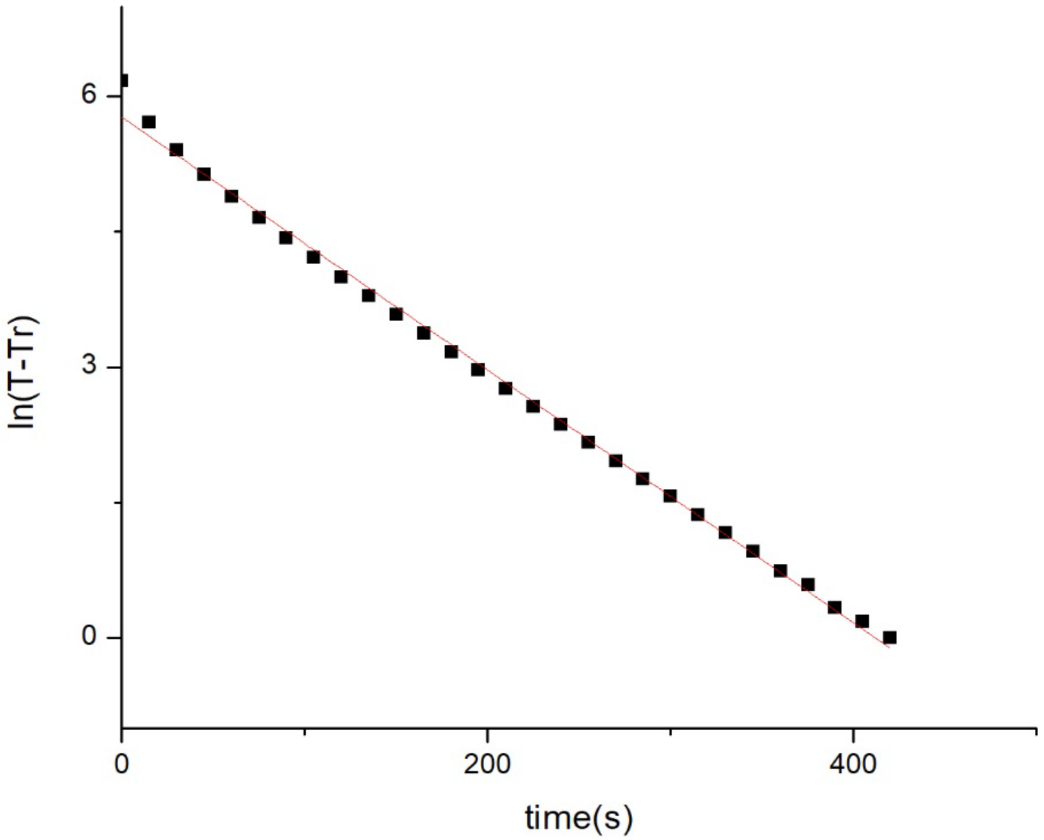
$\boxed{\text{IN THEORY THE FUNCTION IT IS EXPONENTIAL}}$

$$\Delta(T - T_r) = -\frac{H_c}{C \cdot m}(T - T_r)\Delta t \qquad \lambda = \frac{H_c}{C \cdot m} \longrightarrow (T - T_r) = (T_0 - T_r) \cdot e^{-\frac{H_c}{C \cdot m} \cdot t}$$



$y = Ae^{\pm Bx} \longrightarrow \ln y = \ln A \pm Bx.$

www.xuru.org/rt/lr.asp



$y = kx + n$ $k = -0.01402$ and $n = 5.77131.$

coefficient of determination *R*²

$$R^2 = 1 - \frac{SS_{res}}{SS_{tot}}$$

$$R^2 = 0.996808$$

https://www.danielsoper.com/statcalc/calculator.aspx?id=25)

99.66%